



K21U 4551

Reg. No. :

Name :

V Semester B.Sc. Degree CBCSS (OBE) Regular

Examination, November 2021

(2019 Admn. Only)

CORE COURSE IN MATHEMATICS

5B06 MAT : Real Analysis – I

Time : 3 Hours

Max. Marks : 48

PART – A

Answer any four questions. Each question carries 1 mark.

1. For $a, b \in \mathbb{R}$ if $a + b = 0$, then prove that $b = -a$.

2. Find the supremum of the set $\left\{1 - \frac{1}{n} : n \in \mathbb{N}\right\}$.

3. Show that $\lim_{n \rightarrow \infty} \frac{1}{n} = 0$.

4. Give an example of a discontinuous function on \mathbb{R} .

5. Define sequential criterion for the continuity of a function f on \mathbb{R} . (4x1=4)

PART – B

Answer any eight questions. Each question carries 2 marks.

6. State and prove Archimedean property.

7. Determine the set $B = \{x \in \mathbb{R} : x^2 + x > 2\}$.

8. Let $J_n = \left(0, \frac{1}{n}\right)$ for $n \in \mathbb{N}$, prove that $\bigcap_{n=1}^{\infty} J_n = \emptyset$.

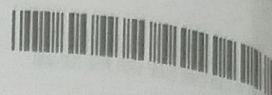
9. Prove that a sequence in \mathbb{R} can have at most one limit.

10. Show that a convergent sequence of real numbers is bounded.

11. Prove that every convergent sequence is a Cauchy sequence.

P.T.O.

K21U 4551



12. If the series $\sum x_n$ converges, then prove that $\lim_{n \rightarrow \infty} x_n = 0$.
13. Check the convergence of the series $\sum_{n=1}^{\infty} \frac{1}{n^2 + n}$.
14. Show that every absolutely convergent series is convergent.
15. Show that the function $f(x) = \frac{1}{x}$ is not bounded on the interval $(0, \infty)$.
16. If functions f, g are continuous at a point c , then prove that $f + g$ is also continuous at c . (8×2=16)

PART – C

Answer **any four** questions. **Each** question carries **4** marks.

17. Prove that the set of real numbers is not countable.
18. State and prove Squeeze theorem.
19. State and prove Bolzano Weierstrass theorem.
20. Let $X = (x_n)$ and $Y = (y_n)$ that converges to x and y respectively, then prove that $X + Y$ converges to $x + y$.
21. Show that $\sum_{n=1}^{\infty} \frac{1}{n}$ is divergent.
22. If $X = (x_n)$ is a convergent monotone sequence and the series $\sum y_n$ is convergent, then prove that the series $\sum x_n y_n$ is convergent.
23. State and prove preservation of intervals theorem. (4×4=16)

PART – D

Answer **any two** questions. **Each** question carries **6** marks.

24. Prove that there exists a positive real number x such that $x^2 = 2$.
25. State and prove Monotone convergence theorem.
26. State and prove D'Alembert's ratio test for series.
27. If $f : [a, b] \rightarrow \mathbb{R}$ is a continuous function, then prove that f has an absolute maximum and absolute minimum on $[a, b]$. (2×6=12)